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Featured Essay

ON STEPHEN P. TOWNSEND'S 1979 PROOF

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Posed in 1950 by Edward Nelson, chromatic number of the plane (CNP) problem lay dormant through the 1950s. Nothing new had been proven after the simple 1950 demonstrations of the CNP's lower bound of 4 by Nelson, and the upper bound of 7 by John Isbell. During the following decade Paul Erdös posed a number of related problems generalizations, but no results followed until Dmitry Raisky surprised the world by proving in his 1970 paper, that the minimal number of colors in coloring of the plane is 4, 5 or 6, if we require each color to forbid a distance (not necessarily the same one). The world would have been surprised even more if it knew that Dima Raisky was a … high school student! (See more about CNP's history and related problems in [Soil] and [Soi2]).

Raisky's paper had inspired a new energy and works by D. G. Larman and C. A. Rogers, and D. R. Woodall. Woodall in his 1973 paper [Woo] re-proved Raisky's result, showed that the rational plane is 2-colorable, and most importantly published a proof that the chromatic number of the plane is 6 or 7 if one requires the monochromatic sets to be closed, or simultaneously divisible into regions (see the definition in [Woo]). The latter was a fine result. However, in 1979 Stephen P. Townsend found an error in Woodall's proof, and constructed a counterexample demonstrating that one essential idea of the proof was false.

This story must remind the readers of the famed Victorian Affair. In 1879 Alfred B. Kempe published a proof of the 4-Color Map-Coloring Theorem, in which 11 years later Percy J. Heawood found an error and constructed a counterexample to demonstrate the irreparability of the hole. Heawood salvaged the 5-Color Theorem, but the 4-Color result had to wait nearly a century for its proof.

Our present story had a better promise for success than its Victorian counterpart, for Townsend went on to prove the Woodall's statement. The promise, however, ran into a wall, when the Journal of Combinatorial Theory's Managing Editor wrote to Townsend on April 3, 1980 as follows:

The Journal of Combinatorial Theory - Series A is now trying very hard to reduce its large backlog, and we ask all our referees to be especially attentive to the question of the importance of the papers. In this case the referee thought that the result was not of great importance. In view of our backlog situation then, we are reluctant to publish the paper. However, since it does correct an error in a previously published paper, we would like to have a very short note about it. Perhaps, you would be willing to do the following: Write a note pointing out the error, stating the theorem (Theorem 1) (without proof) used to get around the trouble, and that the theorem must be used with care to get around the problem.

And so Stephen P. Townsend had satisfied the Editor, and produced a 2-page proofless note [Tow] (he did describe the counterexample in it), which was published the following year. This is where the story was to end in 1981.

No blame may be attributed to Douglas R. Woodall, whose 1973 work remains one of the best works on the subject, and who was first to alert me to his mistake and Townsend's note. "I am a fan of your 1973 paper," I wrote to Woodall in the October 10, 1993 e-mail, in which I called [Tow] "the Townsend's amendment" The following day Woodall replied as follows:

I will put a reprint in the post to you today, together with a photocopy of Townsend's "addendum", as you so tactfully describe it. (The fact is, I boobed, and Townsend corrected my mistake.)

However, regret is in order with regards to the decision by the Editors of the Journal of Combinatorial Theory Series A (JCTA). While they apparently (and correctly) assessed Woodall's paper as being "of great importance" (an impossible test if one interprets it literally), they denied its readers - and the world - the pleasure and the profit of reading the Townsend's proof of the major result, formulated but not proven by Woodall.

We are correcting JCTA's quarter-a-century old mistake today. It gives me a great pleasure to introduce and publish here in *Geombinatorics* the Townsend's proof. In my opinion, it *is* of great importance - judge for yourselves!

Bibliography

[Soil] Soifer, A., Chromatic Number of the Plane & Its Relatives Part I: The Problem & Its History, *Geombinatorics* XII(3), 2003, 131-148.

[Soi2] Soifer, A., Chromatic Number of the Plane & Its Relatives Part II: Polychromatic Number and 6-coloring, *Geombinatorics XII(4),* 2003, 191-216.

[Tow] Townsend, S. P., Every 5-Coloured Map in the Plane Contains a Monochrome Unit, *J. Combinatorial Theory Ser. A 30 (1981),* 114-115.

[Woo] Woodall, D. R., Distances realized by sets covering the plane, *J. Combin. Theory Ser. A,* 14 (1973), 187-200.